

Lösungen Jahresprüfung 4. Klasse Algebra 2017

1. Faktorisieren

$$(a) \quad 2xy - 6y^2 - 4yz = \underline{\underline{2y(x - 3y - 2z)}}$$

$$(b) \quad x^2 - y^2 = \underline{\underline{(x - y)(x + y)}}$$

$$(c) \quad x^2 - 5x - 24 = \underline{\underline{(x - 8)(x + 3)}}$$

$$(d) \quad 6x(y + z) - y - z = 6x(y + z) - (y + z) = \underline{\underline{(y + z)(6x - 1)}}$$

$$(e) \quad x(5y + 5) + (x - 3)(2y + 2) = 5x(y + 1) + 2(x - 3)(y + 1) = (y + 1)(5x + 2(x - 3)) = (y + 1)(5x + 2x - 6) = \underline{\underline{(y + 1)(7x - 6)}}$$

2. Polynomdivision

$$(a) \quad \begin{array}{r} x^3 - 5x^2 + 11x - 10 \\ -x^3 + 2x^2 \\ \hline -3x^2 + 11x \\ \quad 3x^2 - 6x \\ \hline \quad \quad 5x - 10 \\ \quad \quad -5x + 10 \\ \hline \quad \quad \quad 0 \end{array} : (x - 2) = x^2 - 3x + 5$$

$$(b) \quad \begin{array}{r} x^3 + 2x^2 - 3x - 6 \\ -x^3 - 2x^2 \\ \hline -3x - 6 \\ \quad 3x + 6 \\ \hline \quad \quad 0 \end{array} : (x + 2) = x^2 - 3$$

3. Bruchterme

$$(a) \quad \begin{aligned} & \frac{5a + 3b}{3a} - \frac{2a + 5b}{6b} - \frac{8a^2 + 6b^2}{6ab} = \frac{2b(5a + 3b)}{6ab} - \frac{a(2a + 5b)}{6ab} - \frac{8a^2 + 6b^2}{6ab} \\ & = \frac{10ab + 6b^2}{6ab} - \frac{2a^2 + 5ab}{6ab} - \frac{8a^2 + 6b^2}{6ab} = \frac{10ab + 6b^2 - (2a^2 + 5ab) - (8a^2 + 6b^2)}{6ab} \\ & = \frac{10ab + 6b^2 - 2a^2 - 5ab - 8a^2 - 6b^2}{6ab} = \frac{-10a^2 + 5ab}{6ab} = \frac{5a(b - 2a)}{6ab} = \frac{5(b - 2a)}{6b} \\ & = \underline{\underline{\frac{5b - 10a}{6b}}} \end{aligned}$$

$$(b) \quad \begin{aligned} & \frac{3a^2 - 27}{6a + 12} : \frac{a^2 - 6a + 9}{a^2 + 4a + 4} = \frac{3(a^2 - 9)}{6(a + 2)} : \frac{(a - 3)^2}{(a + 2)^2} = \frac{3(a - 3)(a + 3)}{6(a + 2)} \cdot \frac{(a + 2)^2}{(a - 3)^2} \\ & = \frac{3(a - 3)(a + 3) \cdot (a + 2)^2}{6(a + 2) \cdot (a - 3)^2} = \underline{\underline{\frac{(a + 3)(a + 2)}{2(a - 3)}}} \end{aligned}$$

$$(c) \quad \begin{aligned} & \frac{a - b}{(a + b)^2} \cdot \frac{a + b}{a} : \frac{a^2 - b^2}{a^2} = \frac{a - b}{(a + b)^2} \cdot \frac{a + b}{a} : \frac{(a - b)(a + b)}{a^2} \\ & = \frac{a - b}{(a + b)^2} \cdot \frac{a + b}{a} \cdot \frac{a^2}{(a - b)(a + b)} = \frac{(a - b) \cdot (a + b) \cdot a^2}{(a + b)^2 \cdot a \cdot (a - b)(a + b)} = \underline{\underline{\frac{a}{(a + b)^2}}} \end{aligned}$$

4. Bruchtermgleichungen

$$\begin{aligned}
 \text{(a)} \quad \frac{3}{x+1} &= \frac{7}{x-2} \rightarrow \mathbb{D} = \mathbb{R} \setminus \{-1, 2\} \\
 \frac{3(x-2)}{(x+1)(x-2)} &= \frac{7(x+1)}{(x-2)(x+1)} \quad | \cdot (x+1)(x-2) \\
 3x-6 &= 7x+7 \quad | -3x-7 \\
 -13 &= 4x \quad | :4 \\
 x &= \frac{-13}{4} \rightarrow \text{in } \mathbb{D} \rightarrow \underline{\underline{\mathbb{L} = \left\{ \frac{-13}{4} \right\}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \frac{x-8}{x} &= \frac{x}{x+4} \quad | \mathbb{D} = \mathbb{R} \setminus \{-4, 0\} \\
 \frac{(x-8)(x+4)}{x(x+4)} &= \frac{x \cdot x}{x(x+4)} \quad | \cdot x(x+4) \\
 (x-8)(x+4) &= x^2 \quad | \text{TU} \\
 x^2 - 4x - 32 &= x^2 \quad | -x^2 + 4x \\
 -32 &= 4x \quad | :4 \\
 x &= -8 \quad \text{in } \mathbb{D} \rightarrow \underline{\underline{\mathbb{L} = \{-8\}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad \frac{6}{x+5} - \frac{2x+60}{x^2-25} &= -\frac{7}{x-5} \quad | \text{Faktorisieren} \\
 \frac{6}{x+5} - \frac{2(x+30)}{(x-5)(x+5)} &= -\frac{7}{x-5} \quad | \mathbb{D} = \mathbb{R} \setminus \{-5, 5\} \\
 \frac{6(x-5)}{(x+5)(x-5)} - \frac{2(x+30)}{(x-5)(x+5)} &= -\frac{7(x+5)}{(x-5)(x+5)} \quad | \cdot (x-5)(x+5) \\
 6(x-5) - 2(x+30) &= -7(x+5) \quad | \text{TU} \\
 6x-30-2x-60 &= -7x-35 \quad | +7x+90 \\
 11x &= 55 \quad | :11 \\
 x &= 5 \quad \rightarrow \text{nicht in } \mathbb{D} \rightarrow \underline{\underline{\mathbb{L} = \{\}}}
 \end{aligned}$$

5. Gleichungssysteme

$$\begin{aligned}
 \text{(a)} \quad \left| \begin{array}{l} 2x+4y = 2 \\ x+2y = 3 \end{array} \right| \cdot (-2) &\rightarrow \left| \begin{array}{l} 2x+4y = 2 \\ -2x-4y = -6 \end{array} \right| \\
 &0 = -4 \rightarrow \underline{\underline{\mathbb{L} = \{\}}} \\
 \text{(b)} \quad \left| \begin{array}{l} 5x-4y = 6 \\ 8x-7y = 0 \end{array} \right| \cdot \begin{array}{l} 7 \\ (-4) \end{array} &\rightarrow \left| \begin{array}{l} 35x-28y = 42 \\ -32x+28y = 0 \end{array} \right| \quad \begin{array}{l} 70-4y = 6 \\ y = 16 \end{array} \\
 &3x = 42 | :3 \quad \rightarrow \underline{\underline{\mathbb{L} = \{(14/16)\}}} \\
 &x = 14 \\
 \text{(c)} \quad \left| \begin{array}{l} 2x-\frac{5}{3}y = 4 \\ 3x-\frac{7}{2}y = 0 \end{array} \right| \cdot \begin{array}{l} 3 \\ (-2) \end{array} &\rightarrow \left| \begin{array}{l} 6x-5y = 12 \\ -6x+7y = 0 \end{array} \right| \quad \begin{array}{l} 2x-10 = 4 \\ x = 7 \end{array} \\
 &2y = 12 | :2 \quad \rightarrow \underline{\underline{\mathbb{L} = \{(7/6)\}}} \\
 &y = 6
 \end{aligned}$$