

Lösungen Algebraprüfung 4. Klasse - Übungsblatt

a) $x^5 - x^4 = x^4(x-1)$

b) $8x+4 = 4(2x+1)$

c) $x^3+2x^2+x = x(x^2+2x+1) = x(x+1)^2$

d) $4x^2-28xy+49y^2 = (2x-7y)(2x-7y) = (2x-7y)^2$

↳ Man erkennt den Ansatz an den Quadratzahlen 2 & 49.

e) $x^3-3x^2-18x = x(x^2-3x-18) = x(x-6)(x+3)$

f) $x^2(y-3) - x(y-3) - 2(y-3) = (x^2-x-2)(y-3)$

2a) x^2+3x+2

b) $(x^3-x^2-3x-9):(x-3) = x^2+2x+3$

c) $(x^4-3x^3-5x^2+16x-3):(x-3) = x^3-5x+1$

d) x^3-2x^2+x-3

e) $5x^3+2x^2+1$

3a) $\frac{2q^2-4q}{3q-1} + \frac{3-q+q^2}{3q-1} - \frac{4-8q+3q^2}{3q-1} = \frac{2q^2-4q+(3-q+q^2)-(4-8q+3q^2)}{3q-1}$
 $= \frac{2q^2-4q+3-q+q^2-4+8q-3q^2}{3q-1} = \frac{3q-1}{3q-1} = \frac{1}{1} = \underline{\underline{1}}$

b) $\frac{z+1}{z-1} : \frac{z^2+1}{z^2-1} = \frac{z+1}{z-1} \cdot \frac{(z+1)(z-1)}{z^2+1} = \frac{(z+1)^2}{z^2+1}$

c) $\frac{4u-13}{2u(u-1)} - \frac{u-1}{u(u+1)} - \frac{u-3}{(u+1)(u-1)} = \frac{(4u-13)(u+1)}{2u(u-1)(u+1)} - \frac{(u-1) \cdot 2(u-1)}{2u(u-1)(u+1)} - \frac{(u-3) \cdot 2u}{2u(u+1)(u-1)}$
 $= \frac{4u^2+4u-13u-13-(2u^2-4u+2)-(2u^2-6u)}{2u(u-1)(u+1)} = \frac{4u^2+4u-13u-13-2u^2+4u-2-2u^2+6u}{2u(u-1)(u+1)} = \frac{u-15}{2u(u-1)(u+1)}$

d) $\frac{a+b}{a(a-b)} - \frac{a-b}{b(a+b)} + \frac{a(a-3b)}{b(a^2-b^2)} = \frac{b(a+b)^2 - a(a-b)^2 + a^2(a-3b)}{ab(a+b)(a-b)} = \frac{b(a^2+2ab+b^2) - a(a^2-2ab+b^2) + a^3-3a^2b}{ab(a+b)(a-b)}$
 $= \frac{a^2b+2ab^2+b^3-a^3-2a^2b-ab^2-3a^2b}{ab(a-b)(a+b)} = \frac{ab^2+b^3}{ab(a-b)(a+b)} = \frac{b^2(a+b)}{ab(a-b)(a+b)} = \frac{b}{a(a-b)}$

e) $\frac{x^2}{4x(x-1)} \cdot \frac{2(x+1)}{x^3x} - \frac{x(x+2)}{3(x+1)(x-1)} \cdot \frac{x+1}{x} = \frac{(x+1)3}{2x^2(x-1) \cdot 3} - \frac{(x+2) \cdot 2x^2}{3(x-1) \cdot 2x^2} = \frac{3x+1-(2x^3+4x^2)}{3(x-1)2x^2}$

$= \frac{3x+1-2x^3-4x^2}{6x^2(x-1)}$

ist $x-1$ ausklammerbar?

$(-2x^3-4x^2+3x+1):(x-1) = -2x^2-6x-3$
 \rightarrow geht nicht

$$3f) \frac{4a}{(a-b)^2} + \frac{4a^2}{b-a} = \frac{4a}{(a-b)^2} - \frac{4a^2(a-b)}{(a-b)(a-b)} = \frac{4a - (4a^3 + 4a^2b)}{(a-b)^2} = \frac{-4a^3 + 4a^2b + 4a}{(a-b)^2}$$

$$= \frac{-4a(a^2 - ab - 1)}{(a-b)^2}$$

$$4a) \frac{2}{x} + 2 = \frac{10}{x} \Rightarrow x=0 \Rightarrow D = \mathbb{R} \setminus \{0\}$$

HN: x

$$\frac{2}{x} + 2 = \frac{10}{x}$$

$$\frac{2x}{x} + 2x = \frac{10x}{x}$$

$$2 + 2x = 10$$

$$2x = 8$$

$$\underline{x = 4}$$

Kontrolle ✓

$$4b) \frac{x+8}{2x} = \frac{x-4}{6x}$$

$$\rightarrow \begin{cases} 2x=0 \Rightarrow x=0 \\ 6x=0 \Rightarrow x=0 \end{cases} \} D = \mathbb{R} \setminus \{0\}$$

HN: $6x$

$$\frac{6x(x+8)}{2x} = \frac{(x-4) \cdot 6x}{6x}$$

$$3x + 24 = x - 4$$

$$2x = -28$$

$$\underline{x = -14}$$

Kontrolle ✓

$$4c) \frac{x-8}{x-9} = \frac{x-5}{x-7}$$

$$\begin{cases} x-9=0 \\ x=9 \end{cases} \text{ oder } \begin{cases} x-7=0 \\ x=7 \end{cases} \} D = \mathbb{R} \setminus \{7, 9\}$$

HN: $(x-9)(x-7)$

$$\frac{(x-8)(x-9)(x-7)}{\cancel{x-9}} = \frac{(x-5)(x-9)(x-7)}{\cancel{x-7}}$$

$$(x-8)(x-7) = (x-5)(x-9)$$

$$x^2 - 15x + 56 = x^2 - 14x + 45$$

$$\underline{x = 11}$$

Kontrolle ✓

$$4d) \frac{1}{2(x+2)} + \frac{2}{x+2} = \frac{3}{2}$$

$$\begin{cases} x+2=0 \\ x=-2 \end{cases} \} D = \mathbb{R} \setminus \{-2\}$$

HN: $2(x+2)$

$$\frac{2(x+2)}{2(x+2)} + \frac{4(x+2)}{x+2} = \frac{3(x+2)}{2}$$

$$1 + 4 = \frac{3(x+2)}{2}$$

$$5 = 3x + 6$$

$$-1 = 3x$$

$$\underline{-\frac{1}{3} = x}$$

Kontrolle ✓

4e) $\frac{2}{x+2} + \frac{2}{x-2} = \frac{x+3}{(x+2)(x-2)}$ $\left. \begin{array}{l} x+2=0 \\ x=-2 \end{array} \right\}$ oder $\left. \begin{array}{l} x-2=0 \\ x=2 \end{array} \right\}$ $\cdot \text{HN} \rightarrow \mathbb{D} = \mathbb{R} \setminus \{2; -2\}$
 $\frac{2(x+2)(x-2)}{x+2} + \frac{2(x+2)(x-2)}{x-2} = \frac{(x+3)(x+2)(x-2)}{(x+2)(x-2)}$ $\text{HN: } (x+2)(x-2)$
 $2x-4 + 2x+4 = x+3$
 $4x = x+3$
 $3x = 3$
 $\underline{x = 1}$ Kontrolle: ✓

4f) $\frac{5}{x+3} + \frac{2}{x-3} = \frac{x+1}{(x-3)(x+3)}$ $\cdot \text{HN}$ $\mathbb{D} = \mathbb{R} \setminus \{3; -3\}$
 $\frac{5(x+3)(x-3)}{x+3} + \frac{2(x+3)(x-3)}{x-3} = \frac{(x+1)(x+3)(x-3)}{(x+3)(x-3)}$ $\text{HN: } (x-3)(x+3)$
 $5x-15 + 2x+6 = x+1$
 $6x = 10$
 $\underline{x = \frac{5}{3}}$ Kontrolle: ✓

4g) $\frac{4}{x+3} + \frac{12}{x+4} = \frac{12(2x+1)}{(x+3)(x+4)}$ $\mathbb{D} = \mathbb{R} \setminus \{-3; -4\}$
 $\frac{4(x+3)(x+4)}{x+3} + \frac{12(x+3)(x+4)}{x+4} = \frac{12(2x+1)(x+3)(x+4)}{(x+3)(x+4)}$ $\text{HN: } (x+3)(x+4)$
 $4x+16 + 12x+36 = 24x+12$
 $40 = 8x$
 $\underline{5 = x}$ Kontrolle ✓

5a) $\ominus \begin{cases} x - 3y = 15 \\ x + 5y = 11 \end{cases}$
 $2y = 4$
 $y = 2 \Rightarrow x - 6 = 15$
 $x = 21$
 $\underline{\mathbb{L} = \{(21|2)\}}$

b) $\begin{cases} 8x - (3x+19) = 41 \\ 9x - 3x - 19 = 41 \end{cases}$
 $6x = 60$
 $x = 10$
 $\Rightarrow y = 30 + 19 = 49$
 $\underline{\mathbb{L} = \{(10|49)\}}$

c) $\ominus \begin{cases} 3x + 3y = 12 \\ x + 3y = 18 \end{cases}$
 $2x = -6$
 $x = -3$
 $\Rightarrow -3 + 3y = 18$
 $3y = 21$
 $y = 7$
 $\underline{\mathbb{L} = \{(-3|7)\}}$

5d) $\ominus \begin{cases} x - 3y = -19 \\ 3x - y = 23 \end{cases}$
 $-8x = -88$
 $x = 11$
 $11 - 3y = -19$
 $30 = 3y$
 $0 = y$
 $\underline{\mathbb{L} = \{(11|0)\}}$

5e) $\text{mal 3} \rightarrow \oplus \begin{cases} 3x - 7y = 4 \\ -3x + 7y = -4 \end{cases}$
 $0 = 0$
 unendl. viele Lösungen
 $\underline{\mathbb{L} = \{(x|y) \mid 3x - 7y = 4\}}$

5f) $\ominus \begin{cases} 8x - 7y = 31 \\ 2 \cdot 4x - y = 11 \end{cases}$
 $-5y = 9$
 $y = -\frac{9}{5} = -1,8$
 $4x = 11 - \frac{9}{5} = \frac{46}{5}$
 $x = \frac{23}{10} = 2,3$

5g) $\begin{cases} 3 \cdot 35x - 18y = 65 \\ 2 \cdot 25x - 27y = 21 \end{cases}$
 $55x = 165$
 $x = 3$
 $\Rightarrow 75 - 27y = 21$
 $\Rightarrow y = 2$
 $\underline{\mathbb{L} = \{(3|2)\}}$

$\underline{\mathbb{L} = \{(2,3 \mid -1,8)\}}$