

Jahresprüfung 4. Klasse Algebra

Maximal 34.5 Punkte

Faktorisieren

1. Faktorisieren Sie die folgenden Terme: (je 1.5P)

(a) $2xy - 6y^2 - 4yz$

(b) $x^2 - y^2$

(c) $x^2 - 5x - 24$

(d) $6x(y + z) - y - z$

(e) $x(5y + 5) + (x - 3)(2y + 2)$

Polynomdivision

2. Berechnen Sie mit Hilfe der Polynomdivision: (je 2P)

(a) $(x^3 - 5x^2 + 11x - 10) : (x - 2)$

(b) $(x^3 + 2x^2 - 3x - 6) : (x + 2)$

Bruchterme

3. Vereinfachen Sie folgende Terme: (je 2.5P)

(a) $\frac{5a+3b}{3a} - \frac{2a+5b}{6b} - \frac{8a^2+6b^2}{6ab}$

(b) $\frac{3a^2-27}{6a+12} \cdot \frac{a^2-6a+9}{a^2+4a+4}$

(c) $\frac{a-b}{(a+b)^2} \cdot \frac{a+b}{a} : \frac{a^2-b^2}{a^2}$

Bruchgleichungen

4. Lösen Sie folgende Bruchgleichungen. Vergessen Sie den Definitionsbereich nicht. Die Lösung ist als Lösungsmenge zu geben.

(a) $\frac{3}{x+1} = \frac{7}{x-2}$ (2.5P)

(b) $\frac{x-8}{x} = \frac{x}{x+4}$ (2.5P)

(c) $\frac{6}{x+5} - \frac{2x+60}{x^2-25} = -\frac{7}{x-5}$ (3P)

Gleichungssysteme

5. Lösen Sie die folgenden linearen Gleichungssysteme. (je 2.5P)

(a)

$$\begin{cases} 2x + 4y = 2 \\ x + 2y = 3 \end{cases}$$

(b)

$$\begin{cases} 5x - 4y = 6 \\ 8x - 7y = 0 \end{cases}$$

(c)

$$\begin{cases} 2x - \frac{5}{3}y = 4 \\ 3x - \frac{2}{7}y = 0 \end{cases}$$

Lösungen Jahresprüfung 4. Klasse Algebra 2017

1. Faktorisieren

$$(a) \quad 2xy - 6y^2 - 4yz = \underline{\underline{2y(x - 3y - 2z)}}$$

$$(b) \quad x^2 - y^2 = \underline{\underline{(x - y)(x + y)}}$$

$$(c) \quad x^2 - 5x - 24 = \underline{\underline{(x - 8)(x + 3)}}$$

$$(d) \quad 6x(y + z) - y - z = 6x(y + z) - (y + z) = \underline{\underline{(y + z)(6x - 1)}}$$

$$(e) \quad x(5y + 5) + (x - 3)(2y + 2) = 5x(y + 1) + 2(x - 3)(y + 1) = (y + 1)(5x + 2(x - 3)) = (y + 1)(5x + 2x - 6) = \underline{\underline{(y + 1)(7x - 6)}}$$

2. Polynomdivision

$$(a) \quad \begin{array}{r} x^3 - 5x^2 + 11x - 10 \\ -x^3 + 2x^2 \\ \hline -3x^2 + 11x \\ \quad 3x^2 - 6x \\ \hline \quad \quad 5x - 10 \\ \quad \quad -5x + 10 \\ \hline \quad \quad \quad 0 \end{array} : (x - 2) = x^2 - 3x + 5$$

$$(b) \quad \begin{array}{r} x^3 + 2x^2 - 3x - 6 \\ -x^3 - 2x^2 \\ \hline -3x - 6 \\ \quad 3x + 6 \\ \hline \quad \quad 0 \end{array} : (x + 2) = x^2 - 3$$

3. Bruchterme

$$(a) \quad \begin{aligned} \frac{5a + 3b}{3a} - \frac{2a + 5b}{6b} - \frac{8a^2 + 6b^2}{6ab} &= \frac{2b(5a + 3b)}{6ab} - \frac{a(2a + 5b)}{6ab} - \frac{8a^2 + 6b^2}{6ab} \\ &= \frac{10ab + 6b^2}{6ab} - \frac{2a^2 + 5ab}{6ab} - \frac{8a^2 + 6b^2}{6ab} = \frac{10ab + 6b^2 - (2a^2 + 5ab) - (8a^2 + 6b^2)}{6ab} \\ &= \frac{10ab + 6b^2 - 2a^2 - 5ab - 8a^2 - 6b^2}{6ab} = \frac{-10a^2 + 5ab}{6ab} = \frac{5a(b - 2a)}{6ab} = \frac{5(b - 2a)}{6b} \\ &= \underline{\underline{\frac{5b - 10a}{6b}}} \end{aligned}$$

$$(b) \quad \begin{aligned} \frac{3a^2 - 27}{6a + 12} : \frac{a^2 - 6a + 9}{a^2 + 4a + 4} &= \frac{3(a^2 - 9)}{6(a + 2)} : \frac{(a - 3)^2}{(a + 2)^2} = \frac{3(a - 3)(a + 3)}{6(a + 2)} \cdot \frac{(a + 2)^2}{(a - 3)^2} \\ &= \frac{3(a - 3)(a + 3) \cdot (a + 2)^2}{6(a + 2) \cdot (a - 3)^2} = \underline{\underline{\frac{(a + 3)(a + 2)}{2(a - 3)}}} \end{aligned}$$

$$(c) \quad \begin{aligned} \frac{a - b}{(a + b)^2} \cdot \frac{a + b}{a} : \frac{a^2 - b^2}{a^2} &= \frac{a - b}{(a + b)^2} \cdot \frac{a + b}{a} : \frac{(a - b)(a + b)}{a^2} \\ &= \frac{a - b}{(a + b)^2} \cdot \frac{a + b}{a} \cdot \frac{a^2}{(a - b)(a + b)} = \frac{(a - b) \cdot (a + b) \cdot a^2}{(a + b)^2 \cdot a \cdot (a - b)(a + b)} = \underline{\underline{\frac{a}{(a + b)^2}}} \end{aligned}$$

4. Bruchtermgleichungen

$$\begin{aligned}
 \text{(a)} \quad & \frac{3}{x+1} = \frac{7}{x-2} \rightarrow \quad |\mathbb{D} = \mathbb{R} \setminus \{-1, 2\} \\
 & \frac{3(x-2)}{(x+1)(x-2)} = \frac{7(x+1)}{(x-2)(x+1)} \quad | \cdot (x+1)(x-2) \\
 & 3x-6 = 7x+7 \quad | -3x-7 \\
 & -13 = 4x \quad | :4 \\
 & x = \frac{-13}{4} \quad \rightarrow \text{in } \mathbb{D} \rightarrow \underline{\underline{\mathbb{L} = \left\{ \frac{-13}{4} \right\}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{x-8}{x} = \frac{x}{x+4} \quad |\mathbb{D} = \mathbb{R} \setminus \{-4, 0\} \\
 & \frac{(x-8)(x+4)}{x(x+4)} = \frac{x \cdot x}{x(x+4)} \quad | \cdot x(x+4) \\
 & (x-8)(x+4) = x^2 \quad | \text{TU} \\
 & x^2 - 4x - 32 = x^2 \quad | -x^2 + 4x \\
 & -32 = 4x \quad | :4 \\
 & x = -8 \quad \text{in } \mathbb{D} \rightarrow \underline{\underline{\mathbb{L} = \{-8\}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & \frac{6}{x+5} - \frac{2x+60}{x^2-25} = -\frac{7}{x-5} \quad | \text{Faktorisieren} \\
 & \frac{6}{x+5} - \frac{2(x+30)}{(x-5)(x+5)} = -\frac{7}{x-5} \quad |\mathbb{D} = \mathbb{R} \setminus \{-5, 5\} \\
 & \frac{6(x-5)}{(x+5)(x-5)} - \frac{2(x+30)}{(x-5)(x+5)} = -\frac{7(x+5)}{(x-5)(x+5)} \quad | \cdot (x-5)(x+5) \\
 & 6(x-5) - 2(x+30) = -7(x+5) \quad | \text{TU} \\
 & 6x-30-2x-60 = -7x-35 \quad | +7x+90 \\
 & 4x-90 = -7x-35 \quad | +7x+90 \\
 & 11x = 55 \quad | :11 \\
 & x = 5 \quad \rightarrow \text{nicht in } \mathbb{D} \rightarrow \underline{\underline{\mathbb{L} = \{\}}}
 \end{aligned}$$

5. Gleichungssysteme

$$\begin{aligned}
 \text{(a)} \quad & \left| \begin{array}{l} 2x+4y = 2 \\ x+2y = 3 \end{array} \right| \cdot (-2) \rightarrow \left| \begin{array}{l} 2x+4y = 2 \\ -2x-4y = -6 \end{array} \right| \\
 & 0 = -4 \rightarrow \underline{\underline{\mathbb{L} = \{\}}} \\
 \text{(b)} \quad & \left| \begin{array}{l} 5x-4y = 6 \\ 8x-7y = 0 \end{array} \right| \cdot 7 \quad \left| \begin{array}{l} 35x-28y = 42 \\ -32x+28y = 0 \end{array} \right| \quad \begin{array}{l} 70-4y = 6 \\ y = 16 \end{array} \\
 & \rightarrow \left| \begin{array}{l} 3x = 42 | :3 \\ x = 14 \end{array} \right| \quad \rightarrow \underline{\underline{\mathbb{L} = \{(14/16)\}}} \\
 \text{(c)} \quad & \left| \begin{array}{l} 2x - \frac{5}{3}y = 4 \\ 3x - \frac{7}{2}y = 0 \end{array} \right| \cdot 3 \quad \left| \begin{array}{l} 6x - 5y = 12 \\ -6x + 7y = 0 \end{array} \right| \quad \begin{array}{l} 2x - 10 = 4 \\ x = 7 \end{array} \\
 & \rightarrow \left| \begin{array}{l} 2y = 12 | :2 \\ y = 6 \end{array} \right| \quad \rightarrow \underline{\underline{\mathbb{L} = \{(7/6)\}}}
 \end{aligned}$$